

Lecture 12: Work, Energy and Power III

Work/energy conservation

We have seen that when an external force moves a body, it does work given by,

$$W_e = \text{Work by External Force} = \sum_i \vec{F}_i \cdot \Delta \vec{r}_i \quad (1)$$

If the force is a constant, this reduces to,

$$W_e = \vec{F} \cdot \Delta \vec{r} \quad (\text{constant force}) \quad (2)$$

This work can have several different consequences, which can be broken up into various sorts of energy: Kinetic energy; Potential Energy; Dissipated Energy. A fundamental principle of science is that work/energy is conserved and in its general form this is stated as,

$$W_e = \Delta KE + \Delta PE + E_{dissipated} \quad (3)$$

In this part of the course we shall concentrate on the following forms of these energies,

$$KE = \frac{1}{2}mv^2 \quad (4)$$

$$PE(\text{gravity}) = mgh; \quad PE(\text{spring}) = \frac{1}{2}kx^2 \quad (5)$$

$$E_{dissipated} = \mu_k N \Delta r \quad (6)$$

Now the confusing part: Work done by internal forces

In the discussion of Lectures 10 and 11 and to this point in this lecture, I have emphasized the work done by an external force. However in the case of the potential energy and the energy dissipated, there is an opposing force, which is an *internal force*. For example in the case of lifting a mass through a height h , the work done by the external force is mgh . What is the work done by gravity, which is the *internal force* in this case? Since the gravitational force is opposite the external force doing the lifting, the work done by gravity is $-mgh$. The same is true of the other cases as well, for example the work done by the friction force, which is an internal force, has the opposite sign to the energy dissipated, that is,

$$W_f = -E_{dissipated} \rightarrow W_{nc} \quad (7)$$

In the general case where there are several sources of dissipation, they are collectively called non-conservative forces, W_{nc} , which is the term used in the textbook. The bottom line is that in some questions you will need to read carefully to get the sign of the work right. For example the dissipated energy is always positive or zero, so the work done by the friction force is always negative or zero. In contrast, the work done by the gravitational force can be negative (during lifting) or positive (for falling bodies)

Special cases

An important special case is the case where there is no external force, so that we have,

$$\Delta KE + \Delta PE = -E_{dissipated} = W_f; \quad (if \ W_e = 0) \quad (8)$$

A case that is even more special but which is still really important is the case where there is no external force and there is no friction. In that case we have,

$$\Delta KE + \Delta PE = 0; \quad (if \ W_e = 0, \ and \ W_f = 0) \quad (9)$$

Last lecture we used this equation to solve the projectile motion speed problem in a very simple way.

Work as the area under a curve

It is useful to think about work as the area under a curve. To see this consider a one dimensional problem where the force depends on the position, ie the force is $F(x)$. The work is then

$$Work = \sum F(x_i)\Delta x_i \quad (10)$$

By drawing a graph it is possible to see that this is the area under the function $F(x)$. This provides another way of showing that the potential energy stored in a spring is $kx^2/2$.

Power

Power is work per unit time. Note though that we don't usually talk of negative power, even though work itself can be negative. Instead we talk of power dissipated!

An example

A spring with spring constant, $k = 10^8 N/m$ is aligned horizontally and compressed from its equilibrium position by a distance of $x = 1m$. A car body (ie no engine) of mass $m = 400kg$ rests against the spring. The spring is released and the car body rolls at high speed toward a hard brick wall. The car body crashes into the wall and the front of the car crumples a distance of $1m$. a) What is the velocity of the car just before it hits the wall (ignore friction); b) What is the average force exerted by the wall on the car as it crumples; c) What is the average acceleration of the car during the crumpling process; d) what is the average power lost by the car during the crumpling process.